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Solipsism for everyone: principles and theorems

For Zofia, Polish logician of the future

Abstract

Here I represent metamathematically a rigorous and, as I characterize it below, "internalistic," take on solipsism. That take is clearly recognizable as an implementation of central ideas from old-time solipsism and, for it, I demonstrate the following.

1. The Putnam Twin Earth and Wittgenstein Private Language Arguments are ineffective against it.

2. More generally, there can be no deductive refutation of this solipsism employing only premises a committed solipsist would accept: all logically correct derivations from solipsistically true premises lead to conclusions that are solipsistically true as well. Any route to a successful refutation of solipsism must travel via nondeductive inferential paths.

3. The truths that solipsists admit comprise a relatively consistent theory under a reasonable logic and describe a world complete in itself. In other

words, the solipsist can hold a relatively consistent opinion on every circumstance expressible in her language.

4. Every solipsistic theory that is strict – as defined below – and axiomatic is the close translational analogue of an axiomatic nonsolipsistic theory. If the solipsist can axiomatize her nonsolipsistic theories, she can do the same with their solipsistic correlates.

5. A price for all this will be the breakdown in an expected strong connection between the truths of solipsism and what appears to the solipsist. Although, for elementary sentences ϕ, ϕ is solipsistically true just in case it appears to the solipsist that ϕ, this connection cannot plausibly be extended to sentences of her solipsistic language generally. However, the breakdown does bring the solipsist at least one real advantage: she is not, via the strong connection, haunted by the succubus of verificationism.

6. Ontological solipsism does not force upon its adherents Brouwerian intuitionism in mathematics.

7. Lastly, other forms of solipsism, including epistemic and moral solipsism, can be construed along similar lines with similar results. For instance, a strictly epistemic solipsism cannot be refuted on grounds of internal inconsistency unless a correlated nonsolipsistic epistemology is itself inconsistent.

Keywords

appearances, mathematical intuitionism, Private Language Argument, Hilary Putnam, Bertrand Russell, solipsism, translation, Twin Earth, Ludwig Wittgenstein

1. A problem with solipsism and its solution

Presentations of solipsism are sure to tell us – perhaps with annoying insistence – what the solipsist believes not to exist. For example, the solipsist is supposed to demand that such presumably transcendent stuff as God, the soul, and other minds are not real. Worse, these descriptions do not offer us sufficient detail on what the solipsist thinks to exist, apart from intoning that only appearances, or her very own appearances, or her very own sensory appearances, plus maybe her own mind in which to house them all, exist. It is a further and painful disappointment that characterizations of solipsism never seem to explain how to individuate those things that, by solipsistic lights, are exclusively real. Certainly, we need to know when one appearance is the same as or different from another and why such sameness or difference matters. This goes for minds as well, the supposed bearers of appearance. What is it that makes the presumptively *sole* mind in the universe *the solipsist's* specifically? No solipsistic entity without solipsistic identity.

Once such matters get settled, I still must know from the committed solipsist what kinds of things solipsists can say, meaningfully or even truly, about those experiential entities they do allow. (Are there partially as well as wholly committed solipsists? Is there a solipsistic analogue to religious agnosticism?) It seems that, at one point at least in his long career, Bertrand Russell considered a form of agnosticism with regard to the existence of the external world. In Russell (1914a), we find, "[W]e may be urged to a modest agnosticism with regard to everything that lies outside our momentary consciousness. Such a view, it is true, is not usually advocated in this extreme form" (as cited in Trybus 2020: 107–108). After all, the extent of a proposed ontology neither determines nor circumscribes the truths, falsehoods, and solecisms pertaining to items in it. You may recall that German mathematician and logician David Hilbert (1862–1943) staunchly maintained that there are at most finitely many things in the world. Yet, simultaneuously, he held mathematicians readily make meaningful and true pronouncements about infinite collections (Hilbert 1926). No ontology, all by itself, fixes even a proper lexicon and grammar. As if those were not troubles enough, the philosophical literature does not always distinguish solipsism clearly from what it classes betimes as fellow travelers: species of idealism, phenomenalism, and epistemological foundationalism.

I look to put forward a treatment of solipsism, one I call "internal" to the solipsist's speech and thought – rather than "external," one that is as precise as I can make it, and one, I hope, unplagued by various familiar troubles. I shall try to say plainly what, from the standpoint of my imagined solipsist, exists and how these existents must be individuated. As you will see, all entities and identification conditions will both arise from and resolve into appearings. According to Roderick Chisholm (1965: 168), "we can say all we know about perception" in the language of appearances. I go on to describe what the solipsist can say – what a solipsistic theory is – and then explain what it would take for a solipsistic theory to be a theory of a recognizable world.

Apart from a need to make some few obvious and necessary assumptions about the intensional context "it appears to me that ϕ" there is absolutely no demand that, on the present construal of solipsism, knowledge be constructed either wholesale or retail out of appearings (whatever they turn out to be). So, my rendition of solipsism is plainly distinct from, say, a phenomenalism on which physical objects get whipped up from a melange of real or possible appearances, as in Russell (1914b). Also, I never have to say whether appearings are mental or physical, or to answer questions about the essence of mentality or physicality. (I do try to diagnose and treat a dangerously deceptive ambiguity in the term "appearance" and its cognates, though). Along the way, I not only list troubles that my thoroughly modern solipsist will have to face, but also defuse some notable objections laid at her door. One of those seeming troubles is rooted in the solipsist's desire to forge a strong and thoroughly general connection between what she wants truthfully (and solipsistically) to say and what gets through to her via appearance.

My method throughout will be *more geometrico*, that is, Euclidean or Dedekindian in spirit: first setting out clear and justifiable definitions of concepts I deem fundamental, and then deducing from those definitions pertinent results as theorems.

2. Formulating solipsism and problems of externality

There is no one solipsism and there never was. There are many kinds and expressions of them, ranged along varied dimensions. For instance, there are expressions of moral solipsism or extreme egoism. Perhaps one of those is, "I alone am the sole moral agent, and unique source of all to which moral obligation is owed. Morally speaking, I, my designs, my apparent acts, and my preferences are the only moral matters." (Such expressions of solipsism – ones I will term "external" – are hardly unobjectionable, as I am about to show.) Unless one ditches the indexical meanings of the sentences just quoted in favor of their verbal syntax, there is at least one such solipsism for each moral subject. In addition, there are strictly epistemological solipsisms – again, one for each apparent knower, marked out by the demand, "I am the only knower, and I am aware exclusively of my own mind and its contents. All else is illusion, mental construction, or at best supposition." I will not try to ring all the changes; you get the idea. (Of course, if one of these solipsisms were true, all the others would be automatically false.) In the present article, I limit consideration henceforth to expressions, both external and internal, of a narrowly ontological solipsism, leaving all the epistemological, moral, conative, spatial, temporal, and – you name it – other solipsisms behind, including one I noted recently that is perhaps peculiarly American: "while driving the car in heavy traffic" solipsism.

In the philosophical literature, one finds attempt after attempt to formulate ontological solipsism externally. Most are flawed, at times irreparably so. Here is one to be getting on with. As my students say, I found it on the internet.

> The mind is the sole existent. The outside world exists only in the mind of the observer.

This cannot be solipsism. Right off, one wants to know, "Whose mind?" and "Outside of what exactly?" Were descriptors such as "the mind" and "the observer" to imply that there is exactly one mind and one observer, this view would have been true of God before the world was created, when all was without form, and void; and darkness was upon the face of the deep. However, no one conceives of (the possible circumstance presented to us by a) God prior to the act of creation as making the thesis of ontological solipsism true. Alternatively, if these definite descriptions are meant as in Verdi's canzone *La donna è mobile*, then the above is an expression of Berkeleian idealism: maybe lots of minds exist with mental stuff in them and "the mind" refers generally to them all. Third, the sentence "the outside world exists only in the mind of the observer" is a plain oxymoron. (I have to assume that the peculiar phrase "the outside world" has a reasonable meaning. Does a nonphysical mind, which is presumably the sole existent, even have an outside?) I would think that, if an "outside world" exists entirely within the mind of some observer, then it cannot be a truly *outside* world at all. Besides, phrases such as "*X* is only in your mind" are normally used to assert that *X* is wholly fictional, a fantasy. Does a solipsist who accepts the above displayed sentences also hold that, since the mind is the sole existent, even his own pains, for example, are fictional, and never occur? They are, after all, "only in the mind."

A second faulty attempt at externalist formulation derives from a named source, Rudolf Eisler's influential *Wörterbuch* (1904):

> The proprietary I is the only existent. Nothing outside the consciousness of that I exists. (Eisler 1904: 1361)

I presume that such expressions as "the proprietary I" are intelligible, and convey something sensible to someone, if not to me. On that presumption, the two sentences just displayed together imply that the sole existent, this proprietary I dingus, is a strange Möbius-strip or Klein-bottle-like individual, living a hermit's existence entirely alone within its own consciousness, which has to be fully identical to the proprietary I itself. The consciousness-container turns out to be the same as what's contained in it. Even those fascinated by rubber--sheet geometry will agree that no such imaginary circumstance is one in which solipsism, as commonly understood, would be taken to be true. (The reader here begins to see why I take such a lively interest in the identity conditions the solipsist thinks to govern her favored entities.)

The following, third effort exhibits at least the limited virtue of concision. I came across it in the writings of Bertrand Russell.

> I alone exist. (Russell 1923: 191)

This would be true in a scenario familiar to sci-fi fans: after the final cosmic apocalypse, Bertrand Russell remains, if only for a brief time, the sole existent, with body reasonably intact, in an otherwise empty universe. No solipsism here.

The obvious objections equally suffice to shoot down another of Russell's tries at capturing ontological solipsism, namely,

> If [solipsism] is true, it is the assertion that I, Bertrand Russell, alone exist. (Russell 1923: 157)

This peculiar statement plainly entails, now that Russell is long gone, that solipsism is false, not just today but for all eternity thereafter. This seems a burdensome handicap to impose upon any fledgling solipsist struggling to find his or her philosophical way: in order to comply with the view, a particular English nobleman and philosopher has to be resurrected. Surely, in all cases, solipsism is not just a simple matter of one mind rather than another existing alone; it also has to do with what is captured or is capturable in those minds. "Only Bertrand Russell's mind exists and it is thinking constantly and sempiternally and exclusively of pickle relish" is nobody's idea of solipsism. Hopeless expressions, such as one I visited earlier, "the outside world is in my mind," are crazed, last-ditch efforts to cope with this difficulty, I suppose.

I maintain – but have not here proved, to be sure – that any similar attempt to construe ontological solipsism in such an external fashion – from without, so to speak, from outside the single solipsistic perspective – will fail. For one thing, it is not enough for the formulation to speak of minds. Any solipsism has also to impose some manner of constraint upon the mental contents of the sole mind deemed to exist, and on the adequacy of those contents with respect to some standard. The statement "there is only one mind and it thinks constantly of pickle relish" is hardly an adequate expression of the target notion. The solipsist means her ontology to be adequate in that there can be no apparent (to her) difference between the world of the solipsist and any other reasonable conception of world. According to the solipsist, it is impossible to demonstrate by reason or sense that the solipsistic ontology is in any respect lacking.

Moreover and more to the point, the aforementioned attempts at formulation, along with others of the same ilk, will fail in the ways in which parallel externalist attempts in philosophy of mathematics fail. Philosophers of mathematics have tried time and again to define such concepts as "number" from lexical points-of-view outside the realm of mathematics. They sought to define "number" in terms of multitudes, arrays, repetitive mental acts, physical or mental processes of counting-off, and lots of others, including classes of physical or mental items. However, piles of rocks and stacks of wood are not necessary existents; numbers and their interrelations are. Therefore, counterfactual counterexamples undermine immediately any naive insistence that numbers are concrete batches of materials things or conglomerates of them. All three-membered clusters of things mental or physical could cease to exist while the number three goes on subsisting merrily – with its familiar noetic nonchalance (cf. Hambourger 1977).

In much the same fashion, human individuals, their minds, and even events or conditions in those minds are not connected by necessity to the contents of the minds, to what may appear. Hence, an adequate formulation of "solipsism" cannot restrict itself to humans, minds in general or specifically, and occurrent mental stuff, any more than a definition of "film" can be given entirely externally – with respect to projectors and laser-read discs spinning in sync with them, and without reference to what is captured on those reels or discs. Likewise, *Pickwick Papers* is not merely a sequence of printed sheets sewn between two cardboard covers. The externalist about solipsism restricts himself to attempting to characterize solipsism by describing, as if from a God's eye view, the mental equipment of the solipsist, thus ignoring the highly pertinent fact that the universe of the solipsist can be available only internally – in the appearings of the solipsist to herself. As before, counterfactual counterexamples undermine immediately any naive insistence that solipsism consists in the existence of a single mind or person, without reference to *precisely what appears* in the appearings of that person. One can always imagine that one mind empty or that one person existing but thoughtless.

The representation of solipsism I construct in the next few sections will offer some plain advantages over other statements of a generally solipsistic view. First – as we shall see – its formulation does not necessarily entail either that the solipsist is the only existent, that she is the only conscious subject, or that ghostly philosophical revenants like "the proprietary I" exist. Second, I am not obliged, preliminary to defining solipsism, to work out what a mind is, what things would be like were only one mind extant, and what it would mean for such a lonely mind-thing to be uniquely someone's – to be *a particular someone's* mind – even when it is, by assumption, the only mind within a universe that is nothing but a mind.

By the way, there is no conceptual connection apparent to me between my use here of the paired terms "internal" and "external" and debates in semantics and the philosophy of mind between internalists and externalists over the contents of mental states. The present representation of solipsism favors neither side in that debate. I took my use of those terms from topos theorists in mathematics who rightly distinguish between the internal (generally intuitionistic) mathematics of a topos and its external mathematics, which is not necessarily intuitionistic.

3. Two formal languages for solipsism

As mentioned, I cannot boast a firm intellectual grip on such expressions as "the proprietary I," "the metaphysical subject," "the outside world," and "pure appearance." In place of suchlike high-falutin' lingo, my attempt at capturing with all due precision the abstract idea of ontological solipsism, tracing its outer logical contours, if you will, assumes the existence of two formalized languages, one reasonably ordinary and one solipsistic in its intended semantics.

3.1. The idiolect

First, there is the solipsist's own idiolect, the public language $\mathcal{L}^1$ she herself speaks and understands. I imagine $\mathcal{L}^1$ regimented reasonably to the extent that names, pronouns, predicates, and various intensional operators such as "it appears to me that" get distinguished, as well as atomic sentential expressions and identities – all demarcated and set off from conjunctions $\wedge$ and negations $\neg$, while the familiar universal quantifier $\forall x$ stands in for its rough informal equivalents. For the sake of certain expressions, I take $\mathcal{L}^1$ to feature the *syntactic* facility for second-order quantifications $\forall P$ and $\exists P$ over subsets of domains. To banish possible confusions, I assume that $\mathcal{L}^1$ does not contain an existence predicate *Ex* as a primitive. (More on existence predicates anon.) Finally, a reasonably standard logic reigns (at least in part) over inferences between sentences in the idiolect so that, for instance, $=$ in $\mathcal{L}^1$ obeys ordinary logical principles for identity. In addition, the usual natural deduction rules for $\wedge$, $\neg$, and $\forall$ hold sway – at a minimum in extensional settings.

I do not think of $\mathcal{L}^1$ as either chained to or requiring references determined by any specific formal or philosophical semantics. Of course, $\mathcal{L}^1$ has naturally a(n informal) semantics: its sentences do say things, but that semantics need not take the form of familiar possible worlds semantics on which the referents of any wide-scope names are, in the jargon, rigid designators across possible worlds, for example. Nor am I obliged to adopt any particular philosophical vision of naming, whether Millian, descriptivist, or other. However, I will say that $\mathcal{L}^1$'s natural semantics can be captured as models or structures $\mathcal{M}$ for $\mathcal{L}^1$. In a suitable $\mathcal{M}$, idiolectic names denote whatever they denote in that structure, and idiolectic predicates carve out subclasses – perhaps indexed – of the domain $|\mathcal{M}|$ of $\mathcal{M}$. As usual, any model $\mathcal{M}$ marks every sentence of $\mathcal{L}^1$ as either true in or false in it. When it comes to a standard model $\mathcal{M}$ for $\mathcal{L}^1$, I fix a structure $\mathfrak{A}$ that represents a cleaned-up version of the solipsist's beliefs: cleaned-up because those beliefs may, in part and on occasion, be unclear or even inconsistent. I emphasize that $\mathfrak{A}$ captures what the solipsist believes were she not a solipsist or were she taking time off from philosophical theorizing. I think of $|\mathfrak{A}|$ as pulling together what the solipsist takes nonsolipsistically to exist – as described in the $\mathcal{L}^1$ – rather than what, in some sense, actually exists. Hence, when interpreted over $\mathfrak{A}$, the quantifiers $\forall$ and $\exists$ range over the items lodged in the solipsist's everyday, nonsolipsistic ontology, which may include intensional entities.

Dear reader, please note that this idiolect, although spoken perhaps by the solipsist alone, is no private language in the overwrought sense of *Philosophical Investigations* (Wittgenstein 1953) and seemingly numberless commentaries on it. Its names and quantifiers are not restricted to either referring solely to or ranging exclusively over the solipsist's own sensations. (Indeed, my current treatment does not entail that there are such things as sensations as conjured up by philosophers – apart perhaps from those run-of-the-mill sensations to which an optician or neurologist refers during diagnosis and treatment.) Also, $\mathcal{L}^1$'s sentences and inferences over them are in no ordinary way strictly private to her. They can be perfectly public. Using that language, she is well able to speak aloud and to make plain statements in the town square, if necessary. Indeed, $\mathcal{L}^1$ captures the (regimented) *parole* she speaks every day.

In the idiolect $\mathcal{L}^1$, there are certain singly intensional sentential contexts that the operator "it appears to me that" prefaces and governs. (Please be aware that occurrences of the pronoun "me" in these cases and throughout, since they feature in the solipsist's idiolect, refer to the solipsist, not the author.) For example,

It appears to me that I am at my desk

contains an example of such context. When it appears to me that ϕ, it comes to me that ϕ, or the circumstance

$$\langle \text{that } \phi \rangle$$

arises before my faculties in a way redolent of ϕ. I do not want to say that, in the best cases, if it appears to me that ϕ, then it merely *seems* to me that ϕ. Whenever it appears to me that ϕ in the sense here intended, $\langle \text{that } \phi \rangle$ is before my mind and I take it that ϕ. Of course, a sensory appearing apprizing me that f is an increasing mathematical function, for example – perhaps the appearance conveyed by a rising line graph drawn on the blackboard in a mathematics class – need not mirror f in every respect; it need not be infinite in apparent extent, e.g., even though the graph of f is itself infinite. By the way, I do not presume that, if it appears to me that ϕ, ⟨that ϕ⟩ or an appearing of ϕ always comes before one or more of my five normal senses.

Roderick Chisholm has demarcated the pertinent sense of "appears" very well:

> It is important to note that the terms of the ordinary 'language of appearing' – terms such as 'appear,' 'look,' 'seem,' 'sound' – have a number of different senses and that only one of these senses is applicable in the present context. […] But when, in discussing epistemology or esthetics we employ the language of appearing and say, 'The penny appears to be elliptical from this point,' we *don't* mean to convey that in all probability the penny *is* elliptical or that we have any inclination to believe that it is. (Chisholm 1965: 172)

Incidentally, the present author does not agree with Chisholm (1965) in his reluctance about so-called "theories of appearance." For one thing, the solipsist stands under no universal obligation to draw, from the premise that

> It appears to me that *A* is red,

the conclusion that

> *A* stands in the "appearing-to" relation to me.

Chisholm's critical fire seems directed entirely toward implausible consequences accruing to this inference (Chisholm 1965: 175ff).

In addition, the hoary skeptical "paradoxes" of appearing trouble me not one whit. I have in mind such Platonic puzzlers as this: it follows from "It appears to Socrates that Creon is tall" (when viewed from close up) and "It appears to Socrates that Creon is short" (when seen from afar) that there is something oxymoronically both tall and short. I can dispatch these fallacies for good by decorating the "it appears to me that" operator with variables for such appearing-relevant indices as time, distance, orientation, wearing-sunglasses-or-not, and so forth. Subsequently, those indices, if employed, would get to name semantical dimensions, lines of logical longitude and latitude, that feature in models $\mathcal{M}$ for $\mathcal{L}^1$. But, this time, I will not be complicating existing notations by dangling indices off them. It suffices to know, friends and countrymen, that one could.

My formal shorthand for

It appears to me that ϕ

will be

$$A\,\phi.$$

At the present stage, I wish to leave TBA the precise logical or material relations between $A\,\phi$ and A. In particular, I do not assume from the outset that appearings as recorded by true idiolectic sentences of the form "$A\,\phi$" are either always veridical, i.e., that

$$A\,\phi \rightarrow \phi, \text{ for all } \phi,$$

or always complete, i.e., that

$$\phi \rightarrow A\,\phi, \text{ for all } \phi.$$

These and moderated versions of them get introduced and examined as explicit assumptions – piecemeal and as needed – for some theorems on solipsism I prove below. Such kindred and more-or-less dubious assumptions as commutativity of A with logical connectives, e.g., for any ϕ,

$$A\,\neg\phi \text{ if and only if } \neg A\,\phi,$$

will be introduced analogously, handled gingerly, and made only as required.

At the moment, I imagine taking on, regimenting, and attaching suitable logical forms to the statements in the solipsist's *entire* everyday idiolect $\mathcal{L}^1$ – all at once. For the treatment of solipsism to come, however, such wholistic regimentation is nowise required. One could instead attempt temporary, local, or patchwork solipsism(s), ones starting with formalizations not of the whole but of reasonably rounded fragements of the idiolect, say axiomatizations of the mathematics known to the solipsist, or of her verbal reaction to a specific perceptual experiment or series of such experiments. (Such experiments and the solipsist's reports on them afford the informal bases for some epistemic models I construct in section 11, below.) For instance, the work here does not demand a "once and for all" sorting of atomic from non-atomic statements in $\mathcal{L}^1$. One might consider some idiolectic statements as atomic for a certain, restricted analytical purpose, statements that otherwise would be said to have complex logical forms featuring multiple, even deeply nested quantifiers and connectives.

3.2. The solipsist's language: words and solipsistic objects

The proprietary first-order language of solipsism *in se ipso* is $\mathcal{L}^2$. Its basic equipment is drawn entirely from what appears. Why are two formal languages necessary to map the rough terrain of solipsism? Because one cannot always take the solipsist strictly at her word. If she says, "horse," and is consistent as a solipsist, she will mean by that word "apparent horse," i.e., "what appears to her a horse and is of a nature set by her appearings." Were she to say, "The horse runs," she will presumably mean, "It appears to me that the apparent horse runs." Incidentally, nothing in the current treatment condemns me to reifying appearances. Hence, I need not infer from "Black Beauty is an apparent horse" to "The title 'Black Beauty' names an appearance."

I tend to write, therefore, of appear*ings* rather than appear*ances* – an *aide mémoire* to my readers.

$\mathcal{L}^2$ is defined over $\mathcal{L}^1$ and contains, for each primitive predicate R and connective or first-order quantifier of the latter, its own version of it, which I denote using the same symbol as that for its $\mathcal{L}^1$-correlate; there will be no confusion, I promise. Indeed, I can, if I wish, imagine $\mathcal{L}^2$ to be a first-order sublanguage of $\mathcal{L}^1$, except that it boasts the existence predicate Ex as well. [For more detail on languages and logics with explicit existence predicate, v. Scott (1979).] To the solipsist, all that she countenances solipsistically has in some form to appear. Therefore, $\mathcal{L}^2$ does not contain the A operator for "it appears to me that," because everything to be reported there is *already* appearing-to-her, at least *in nuce*. It is in the language $\mathcal{L}^2$ that the solipsist lays out what it is to exist solipsistically, what individuates those things she – as solipsist – deems exist, and what she wishes, in her official capacity, to say about them.

To capture the meanings the solipsist expresses, I define a translation from $\mathcal{L}^2$ into $\mathcal{L}^1$ via the transformation $\phi \mapsto \phi^s$. In that definition, τ and σ are terms – variables or names – of both languages.

Definition: With ϕ and ψ from $\mathcal{L}^2$,

1. $(E\tau)^s = \exists P(A\ P\tau) \wedge \forall P[A\ P\tau \leftrightarrow \exists x(x = \tau \wedge A\ Px)]$.
2. $(\tau = \sigma)^s = \forall P[A\ P\tau \leftrightarrow A\ P\sigma]$.
3. For any atomic formula Rxy other than Ex and $=$,
$$(R\tau\sigma)^s = A\ R\tau\sigma.$$
4. $(\phi \wedge \psi)^s = (\phi)^s \wedge (\psi)^s$.
5. $(\neg\phi)^s = \neg(\phi)^s$.
6. $(\forall x\phi(x))^s = \forall x((Ex)^s \rightarrow (\phi(x))^s$.

The first clause explains that what exists for the solipsist is simply what features in appearings and persists through varied appearings. In logical terms, what is said in $\mathcal{L}^2$ to exist is picked out by a name or variable that takes wide scope over all idiolectic contexts of the form $A\ \phi(\ldots)$. By "takes wide scope," I mean that the conditional with antecedent

$$A\ \phi(a)$$

and consequent

$$\exists x(x = a \land A\ \phi(x))$$

is one the solipsist would accept, together with its converse. In other words, a term τ takes wide scope across the "appears to me that" operator when the solipsist is willing to quantify into τ's place from outside the context. For adherents to such gospels of late analytic philosophy as Quine (1966), I am exploiting his famous distinction between "It appears to me that someone is a spy" and "Someone is such that it appears to me that she a spy," although in hardly a fashion that would have pleased ole Quine. For instance, the name "Natasha" in the idiolect gets wide scope in the relevant context if, from "It appears to me that Natasha is a spy," the solipsist is willing to conclude that "Someone, Natasha in particular, is such that she appears to me a spy," and conversely. It is essential, dear reader, to keep firmly in mind that these "appears that" contexts and inferences on them are always from the vantage of the "me," the first person: they are self-ascriptions.

Please remember also that, in the standard model $\mathfrak{A}$ of $\mathcal{L}^1$, the existential quantifier ∃ ranges over what the solipsist believes, nonsolipsistically speaking, to exist, which may be an intensional entity, rather than what exists in reality and independently of her beliefs. Therefore, when a name that comes up in one or more of her reports on appearings takes wide scope, she believes the bearer of that name, though given in her appearings, to subsist apart from any particular appearing in which it features.

Inter alia, this condition on existence removes from existence-bearing consideration complements in such "appears to me that" constructions as the Macbeth-sentence "It appears to me that *a* is a dagger which I see before me, the handle toward my hand," wherein the name "*a*" denotes the famous but illusory dagger. For Macbeth realizes that nothing appears to him as a dagger. Therefore, he would not infer from the truth of

It appears to me that *a* is a dagger which I see before me

to

∃*x*(such that $x = a$ and it appears to me that *x* is a dagger which I see before me).

Therefore, a solipsistic Macbeth would not admit the apparent dagger to the catalogue of his ontology.

One might capture René Descartes's famous *cogito* (Descartes ed. 1979) in the present terms as an argument for taking the personal pronoun "I" always to have wide scope across "appearing that" contexts. Descartes can be thought to be reasoning, in his second *Meditation*, from the premise

It appears to me that I think

to

There is something *x* such that $x =$ I and it appears to me that *x* thinks.

Perhaps this inference was to be mediated, according to Descartes, by the extra premise that

If it appears to me that I think, then I indeed am thinking.

The second of the clauses in the above definition,

$$(\tau = \sigma)^s = \forall P[A\ P\tau \leftrightarrow A\ P\sigma],$$

guarantees that the solipsist's existents not only arise from appearing via persistence, but are also individuated by appearings. As we have demanded, individuating solipsistic existents is the sole prerogative of appearance: what an item is, where it begins, and where it leaves off. Appearing determines it all – entirely what it is to be a single item. For *a* and *b* satisfying the right-hand side of the second clause,

$$\forall P(A\, Pa \leftrightarrow A\, Pb),$$

the solipsist is willing to reason from

It appears to me that $\phi(a)$

to

It appears to me that $\phi(b)$,

and conversely. Under such circumstances, *a* and *b* are identical solipsistically. In addition, the second clause ensures that the relation $\lambda x \lambda y(x = y)^s$ has logical properties requisite to equality: reflexivity, symmetry, transitivity, and substitutivity through all contexts $A\ \phi$.

The view here set out is plainly a form of ontological solipsism in that those things that exist are only those that appear to the solipsist to exist, and they are individuated according to their appearings to the solipsist. However, it is not the kind of methodological solipsism (cf. Goodman 1951) according to which the solipsist is obliged somehow to reconstruct, using relations or other logical devices, from contents of appearings, entities whose existence is not "given in immediate experience," as was said. Nor need it be the latter-day methodological solipsism on which the appearings (and other such states) of the solipsist must be individuated narrowly as in Fodor (1980). Moreover, the present solipsism *need not* (but betimes may!) be as metaphysically draconian as other forms of the doctrine, should the solipsist so decide. For example, in the solipsist's idiolect, the term "John," as it refers to the solipsist's brother, as well as the word "I" as applied to herself, can both be granted wide scope. From "it appears to me that John is a person," the solipsist may be perfectly ready to infer "There is someone, John by name, who appears to me a person." As we have seen, similar inferences may accompany "I". So, she and other apparent persons continue to exist solipsistically, except that any properties they have are determined entirely by her own appearings.

As emphasized above, this representation of solipsism is free of the metaphysical ghosties and specters that haunt traditional renderings of the idea. There need be no private languages, no pure appearances, no sense-data, no (nonclinical) sensations, no epistemological or metaphysical subjects. There need be no disembodied minds, unless they begin to appear to my thoroughly modern solipsist, of course. By the solipsist's lights, there need be no God, should the solipsist insist that He fails altogether to appear or appears merely as Macbeth-like hallucination.

The third clause guarantees a good measure of the strong connection a solipsist desires between her appearings and her solipsistic truths. For her, if she takes "John is tall" to be an atomic predication, it is true that John is tall in $\mathcal{L}^2$ just in case "it appears to me that John is tall" is true in her idiolect $\mathcal{L}^1$. What I am calling "the strong connection" is or should be a persistent feature of solipsism: that everything the solipsist thinks to be true must be drawn from and resolvable back into appearings. As we shall see, it is one of the present results that my solipsist has some trouble maintaining a general strong connection in tandem with other logically desirable properties of her view.

Again, I remind the reader that I could decorate the $A\ \phi$ notation with indices to avert any old-time skeptical problems arising from the relativity of appearance. Hence, the solipsist of my construction is untroubled by the possibility that it appears to her that the sky is blue (when viewed with the naked eye) and so is blue (solipsistically), while it appears to her that the sky is green (when viewed wearing sunglasses) and so is green (solipsistically).

The fourth and fifth clauses above enforce the demand that the propositional logical connectives of the solipsistic language $\mathcal{L}^2$ are precisely those of $\mathcal{L}^1$ and bear the same fundamental logical powers.

The sixth clause tells us that the solipsist always quantifies exclusively over those things that are revealed in appearings and yet are relatively independent of them, i.e., the dinguses that satisfy *Ex*.

The current line of approach to solipsism is not wholly unprecedented in the literature. It stands in closer family resemblance to certain other expressions of solipsism, more considered versions, I believe, than those examined earlier. One of these Russell announced on page 191 of his 1923. There, he offered his readers the *aperçu*, "data are the whole universe" as a principle for solipsism, wherein the term "data" is to be defined by simple, direct enumeration of the contents of all data points. As I emphasized, it is the data-contents rather than media or means of storage, i.e., what the book says rather than what paper, glue, and cardboard covers comprise it physically, that Russell had in mind when writing "data are the whole universe." A strenuous and detailed effort at axiomatizing solipsism is the centerpiece of Todd (1968). Todd claims both Carnap (1928) and Goodman (1951) as forebears. In this connection, philosophers Christine Ladd-Franklin (Trybus 2020) and Richard von Schubert-Soldern (1906) deserve mention. Both of them grasped, early on, the import of what I have termed "internalistic" developments of solipsism.

4. Identity, solipsism, and a theorem

Under weak and not unreasonable conditions on "it appear to me that," the second of the above clauses becomes a theorem of the solipsistic system rather than a fundamental principle of translation.

Theorem: Assume in $\mathcal{L}^1$ that both

$$A\, a = a$$

and,

$$A\, a = b \rightarrow a = b$$

obtain. Then, if $(Ea)^s$, and $(Eb)^s$ hold there, so does

$$a = b \leftrightarrow \forall P(A\, Pa \leftrightarrow A\, Pb);$$

in other words,

$$a = b \leftrightarrow (a = b)^s$$

is true.

Proof:

For the left-to-right direction, assume that $a = b$ and $A\, Pa$. By $(Ea)^s$,

$$\exists x(x = a \wedge A\, Px).$$

Since = in $\mathcal{L}^1$ is assumed to obey its familiar laws,

$$\exists x(x = b \wedge A\, Px).$$

By $(Eb)^s$, $A\, Pb$ holds. Hence,

$$A\, Pa \rightarrow A\, Pb.$$

The proof of

$$A\, Pb \rightarrow A\, Pa$$

is analogous.

Please note that, for this half of the theorem, the solipsist need impose no special requirement upon the "it appears to me that" context. If x and y exist solipsistically, their idiolectic identity = guarantees their solipsistic identity $(x = y)^s$.

For the right-to-left direction, assume that $A\ a = a$ and $A\ a = b \rightarrow a = b$. This time, start with

$$\forall P(A\ Pa \leftrightarrow A\ Pb).$$

From $A\ a = a$ and the above display, it follows that $A\ a = b$. From

$$A\ a = b \rightarrow a = b,$$

one obtains $a = b$.

For this half of the theorem, it is not required that a and b exist solipsistically.

The proof of the theorem would have been simplified slightly had a Leibnizian definition of = in $\mathcal{L}^1$, namely,

$$x = y \text{ if and only if } \forall P(Px \leftrightarrow Py),$$

been adopted, rather than leaving = a primitive.

5. Objections overturned

Some philosophers have maintained that both Putnam's Twin Earth Argument and Wittgenstein's Private Language Argument are fatal to solipsism. My contention in this section is that, even if these ideas may be effective antidotes to some traditional formulations of solipsism, they are complete flops when confronted with solipsism in the current vein. Before I turn to examining each argument individually, I have to remind you of something. If either argument is to succeed against solipsism, the premises of the argument have to be acceptable to the solipsist. Simply showing that solipsism is wrong by deploying premises that no solipsist, old-time or other, would grant is no philosophical achievement. To refute solipsism without a constraint on the premises, all it would take – in the case of traditional solipsism at least – is the casual remark that there is more than one person and more than one mind, and that some things are not represented as mental contents. This would show – but only to a non-solipsist – that any solipsist who maintains there to be just one mind is wrong. Like ministers, philosophers should not preach to the choir (cf. Moore 1925, 1939).

5.1. The Twin Earth Argument

According to Hilary Putnam (1975), it is possible that on "Twin Earth" – a counterfactual world remarkably similar to the actual one – there is a water-like substance associated there with all the usual, in-our-world experiences of water. This alternative stuff – for our purposes here, call it "twater" – appears to all and sundry Twin Earthians just as good old water does to us. Among other things, twater is clear, wet, refreshing when consumed on hot days, and fills lakes. Twin Earthians even call it "water." It is indistinguishable, yet subtly and unnoticeably different from ordinary water: twater has a distinctive chemical composition, say XYZ, rather than H_2O. Twater looks and seems just like, but is not, water.

Putnam draws from his *Gedankenexperiment* the conclusion that "meanings [by that, he means reference-determiners] ain't in the head." For what the Twin Earthian thinks and believes and thinks to see in the face of twater are, in Putnam's imagination, precisely what an Earthman or Earthwoman thinks and believes and thinks to see in the face of Earth-bound water. But what the Twin Earthian thinks and talks *about*, what he or she refers to with those thoughts and sayings, is twater, different from what the Earthperson thinks and talks *about*: water. What determines the reference of the word "twater" on Twin-Earth cannot be the appearings of twater, but must be twater itself. What determines the reference of "water" around here cannot be appearings of water, but must be water itself. So, if Putnam is right, the meanings of terms the solipsist (for instance) employs cannot be determined by such mental entities as his or her own appearings – things that are entirely "in the head." (By the way, this "in the head" lingo has all the flaws and more of those locutions traditionally employed to describe solipsism externally, e.g., "outside world." The present author repeats the catchphrase, but hardly endorses it. As you have seen, for fleshing out our solipsism, no final statement on the nature or natures of appearances need be made.) Therefore, since the solipsist can, it seems, speak with meaning and think coherently about water – as opposed to twater, there must be something that is not "in her head," is not resolvable into her appearings, is not mind or mind-stuff, but determines her meanings, makes her thoughts and sayings be about water, not twater. Hence, solipsism is (supposedly) false. Thus, Putnam.

Undeniably, the statement forming the crux of the Putnamian objection to solipsism is, specifically,

> There is a substance distinct from water but sharing all its appearances.

If this is demonstrated to the satisfaction of the solipsist, she and her view are sunk. As should already be plain to you from the two sections foregoing, my solipsist acknowledges and individuates her entities strictly according to her own appearings.

Apart from complaining that appearings may not be "in the head," my solipsist has two obvious and successful lines to follow in response to Putnam's contentions about Twin Earth and its twater. Concerning the above crucial statement, there are two dialectical possibilities. On the one hand, that statement could be a premise of Putnam's argument. In that case, the solipsist has merely to reject it: to her thinking, there simply are no such substances. This time, Putnam – or an antisolipsistic objector who adopts the Twin Earth fantasy – has simply begged the question. On the other hand, were that same statement, the one just displayed, a conclusion to Putnam's vision of Twin Earth and its twater, the argument would be plainly fallacious. All that the musings about imaginary, possible worlds are able to show is that

> It is imaginable that there be a substance distinct from water but sharing all its appearances.

Clearly, the crucial statement, the first of the two statements just displayed, the one by which the solipsist is to be confounded, does not follow from this last. The solipsist may well grant that such a circumstance is imaginable, but may also deny that it ever actually occurs. The scenario of Twin Earth does not show it to occur, and the argument in which it is employed is, therefore, a *nonsequitur*.

5.2. The Private Language Argument

I will now run through three historically notable takes on the (so--called) Private Language Argument, the *fons et origo* of the three being remarks 243 and following in Wittgenstein's *Philosophical Investigations* (Wittgenstein 1953). In each case, the solipsist can mount a successful defense.

First, there is the Private Language Argument as some commentators construed it in the 1960s and 70s. Here is Antony Kenny, writing during that era, on the concept of private language.

> A private language, in the sense discussed by Wittgenstein, is a language whose words 'refer to what can only be known to the person speaking:

> to his immediate private sensations' (Wittgenstein 1953 § 243). (Kenny 1973: 179)

This is a sort of language that Wittgenstein's linked remarks were supposed to have ruled out of existence. Wittgenstein *epigoni* of the day maintained a principal conclusion of the argument to be roughly,

> There is no language whose referring terms are restricted to the sensations of a single individual. (cf. Wittgenstein 1953: § 258ff)

The solipsism constructed in the present essay is in no way open to the objection that either $\mathcal{L}^1$ or $\mathcal{L}^2$ is a language whose terms denote the sensations of the solipsist exclusively. For one thing, the solipsist's appearances are not limited to her sensations; her appearings are not all and only sensings. Like most of us, she could claim nonsensory introspective appearings. For example, she may well insist that it often appears to her that she is thinking, and that she is aware of this fact on the basis of appearings independent of the five canonical senses. Moreover, she is aware of the relative positions of her own feet under her writing table thanks to nonsensory appearing, even though she is neither looking at nor touching her feet or legs.

By the way, the solipsist's response to this first version of the Private Language Argument is premised on the assumption that the commentators in question were clever enough to sieve a decent, valid argument out from the tangled thought-bolognese of Wittgenstein (1953). As far as I can tell, they never managed that.

For another thing, if John is the solipsist's brother, the solipsist might be willing to infer from

> It appears to me that John is my brother

to

There is someone, John, who appears to me to be my brother,

and similarly for other of John's appearings. In consequence, she could maintain as well that it appears to her that John is human. If so, she could demand, in her solipsistic language $\mathcal{L}^2$ and in accord with her principles as set out in the translation scheme above, that John exists and that he is human. Therefore, her solipsistic language would contain a term, viz., "John," that does not refer to any of her own sensations, but to a human, solipsistically understood.

A second interpretation of the Private Language Argument has it that there can be no language spoken by a single speaker only. The existence of a proper language requires that, given any one speaker *S*, there are other speakers capable in principle of checking up on and correcting the speech of *S* (cf. Wittgenstein 1953: § 270). Such a bald claim – no language can have only one speaker – is obviously empirically false. There are a number of indigenous Native American languages spoken nowadays by only a single remaining person. This patent fact never seems to have bothered Wittgenstein interpreters of this second sort. Any argument leading to this false conclusion has itself to be fatally flawed. To this interpretation of the argument, my solipsist can further reply along these lines: it certainly appears to her that her existing brother John sometimes corrects her speech, and that suffices to guarantee that John indeed corrects her speech, solipsistically understood.

Third, Kripke's resuscitation of the Private Language Argument (Kripke 1982) suggests an objection to solipsism along mathematical lines. One can formulate that objection as follows.

> The solipsist may allow that there are numbers, because she may decide, for instance, that it is a consequence of 'it appears to me that three is the number of Stooges' that 'there is a number, namely three, that appears to me the number of Stooges.' Much the same goes for mathematical entities of more sophisticated varieties, among them algebraic, irratio-

nal, and real numbers. However, it derives immediately from the second clause of the translation $\phi \mapsto \phi^s$ that, since the stuffs the solipsist will countenance are individuated by appearings, there can be no more distinct objects in her solipsistic ontology than there are appearings of objects. In the patois of mathematicians, her set of objects is a quotient, via the equivalence relation

$$x \sim y \text{ if and only if } \forall P(A\ Px \leftrightarrow A\ Py),$$

of her set of appearings. This means that, if she has or has had at most finitely many appearings recorded by at most finitely many true sentences in $\mathcal{L}^1$ of the form '$A\ \phi$', there can be at most finitely many objects in her ontology altogether.[1]

The Dedekind–Chomsky Argument, remodeled for solipsism, overturns this objection. Richard Dedekind (1888) used one rendition of the argument to show that there are infinitely many objects of thought. Noam Chomsky (1968) used another to argue that there are infinitely many sentences of English, all of which native speakers of English have under their semantic competence. The solipsist can steal a page from Dedekind and Chomsky, and argue along parallel lines: 0 exists and appears to her an existing natural number. Furthermore, for each specific natural number x, if x exists and appears to her a natural number, then $x + 1$ likewise exists and appears to her a natural number. She accepts the conditional "if x is a natural number, so is $x + 1$." Remember: she treats the connectives and relativized quantifiers as we all do! By Dedekind's definition of infinite

[1] In his *Theory of Knowledge* (ed. 1992), Russell offered an argument against solipsism – employing the infinity of prime numbers – in spirit very like this one. He there argued: "It seems certain that we shall not think of more than a finite number of arithmetical facts in the course of our lives, and we know that the total number of arithmetical facts is infinite. […] It is therefore certain that there are mathematical facts which do not enter into our total experience" (Russell ed. 1992: 13).

collection, she can then claim that there are an infinite number of existing natural numbers. Therefore, natural numbers exist, and there are infinitely many of them, so she countenances infinitely many items in her solipsistic worldview.

6. Solipsism as complete theory

Definition: A sentence ϕ of $\mathcal{L}^2$ is (*solipsistically*) *true* (with respect to model $\mathcal{M}$) – in symbols, $\phi\, \varepsilon\, \mathcal{S}$ – just in case its solipsistic translation ϕ^s holds in the model $\mathcal{M}$ of the solipsist's original idiolect $\mathcal{L}^1$ or

$$\phi\, \varepsilon\, \mathcal{S} \text{ if and only if } \mathcal{M} \vDash \phi^s.$$

The set $\mathcal{S}$ is my chosen rendering of the solipsist's picture of the world. An explicit superscript, e.g., $S^{\mathcal{M}}$, as a reminder of the dependence of S on the particular idiolectic $\mathcal{M}$ model at issue, has been suppressed.

Note: Please remember that the standard model $\mathfrak{A}$ of the solipsist's idiolect is a rounded-off representation of what the solipsist takes to be true under normal, nonsolipsistic conditions of truth and understanding. Hence, the particular $\mathcal{S}$ corresponding to $\mathfrak{A}$ represents the collection of formulae, solipsistically understood, that the solipsist accepts *in propria persona*. It is currently of reduced moment which, if any, of these $\mathcal{S}$-formulae may be true in reality.

Given the translation from $\mathcal{L}^2$ into $\mathcal{L}^1$ preserving as it does the meanings of the standard connectives, the solipsist's concept of deduction $\vdash$ differs inessentially from familiar standards. She has only to mount deductive "guards" (cf. Dennis 2012) of the form *Ex* on the rules for quantifiers. To be perfectly specific, I here think of $\vdash$ over $\mathcal{L}^2$ laid out in Gentzenesque natural deduction format.

Definition: The basic derivation rules for $\vdash$ governing propositional connectives are identical to the usual ones, e.g.,

$$\Gamma \vdash (\phi \wedge \psi) \text{ if and only if } \Gamma \vdash \phi \text{ and } \Gamma \vdash \psi.$$

Only the derivation rules governing the universal quantifier require extra assumptions or *guards* of the form *Ex*. Those rules are

$$\text{If } \Gamma; Ex \vdash \phi(x), \text{ then } \Gamma \vdash \forall x \phi(x),$$

provided that x is not free in Γ, and

$$\text{If } \Gamma \vdash E\tau \wedge \forall x\, \phi(x), \text{ then } \Gamma \vdash \phi(\tau).$$

There is no trouble in proving that $\mathcal{S}$ is a theory or, equivalently, that $\vdash_s$ is sound with respect to solipsistic truth.

Theorem: (Soundness) If $\Gamma \subseteq \mathcal{S}$ and $\Gamma \vdash \phi$, then $\phi\, \varepsilon\, \mathcal{S}$, for all sentences ϕ and sets Γ of sentences from $\mathcal{L}^2$.

Proof: For the sake of this proof, let $\vdash_s$ be solipsistic derivation in $\mathcal{L}^2$, as just defined. Let $\vdash$ stand for the standard derivation relation over $\mathcal{L}^1$. By induction on the derivation trees implementing $\vdash_s$, one proves for Γ and ϕ from $\mathcal{L}^2$ that, if $\Gamma \vdash_s \phi$, then $\Gamma^s \vdash (\phi)^s$.

It follows immediately from the Soundness Theorem that there can be no cogent refutation of solipsism starting from premises the solipsist herself accepts and proceeding according to the deductive consequence ⊢. Anything she accepts is in the relevant set $\mathcal{S}$ of solipsistic truths, and anything that follows logically from those truths must also be true for the solipsist.

Given any model $\mathcal{M}$ for $\mathcal{L}^1$, the corresponding set $\mathcal{S}$ is *negation* or *Post complete*.

Definition: A set Γ of sentences is *Post complete* whenever, for all sentences ϕ of Γ's language, either $\phi\, \varepsilon\, \Gamma$ or $\neg\phi\, \varepsilon\, \Gamma$.

Theorem: (Post Completeness) For each model $\mathcal{M}$ of $\mathcal{L}^1$, the corresponding set $\mathcal{S}$ is Post complete.

Proof: Obvious from the fifth clause in the definition of $\phi \mapsto \phi^s$.

So, there is no possible query expressible in the solipsist's language $\mathcal{L}^2$ for which her theorizing offers in principle no answer whatsoever. There is nothing she is missing. One can therefore describe the Post Completeness Theorem as informing us that, for every model of $\mathcal{L}^1$, the sentences in $\mathcal{L}^2$ solipsistically true relative to that model do indeed describe a world and, crucially, one drawn at bottom directly from appearings to the solipsist.

Note: Please be clear that, for the theorems of this section, no particular informal interpretation or reading of the form $A\ \phi$ has been adopted or imposed. Hence, the solipsist can set the usage of "it appears to me that" ad. lib. and still be sure, for instance, that her solipsism cannot be undermined from within, cannot be refuted by logically correct reasoning from premises she herself would grant. Once again, I encourage the reader to compare these results with G. E. Moore's trenchant (1925, 1939).

7. Theories and solipsistic things

In the present section, "$\vdash$" sometimes refers to derivability in $\mathcal{L}^2$, sometimes to derivability in $\mathcal{L}^1$, and sometimes (as in the definition of "theory," *infra*) to both. Context should disambiguate. With its subscript, "$\vdash_1$" refers to ordinary derivability over $\mathcal{L}^2$ using first-order predicate logic.

Definitions:

1. A T in $\mathcal{L}^1$ or $\mathcal{L}^2$ is a set of sentences of either language entirely that is closed under its proprietary deductive relation $\vdash$, that is, for all sentences ϕ and sets Γ of sentences from $\mathcal{L}^1$ (or from $\mathcal{L}^2$), if $\Gamma \subseteq T$ and $\Gamma \vdash \phi$, then $\phi\ \varepsilon\ T$.

2. If Θ is a set of sentences of $\mathcal{L}^1$, $s^{-1}(\Theta)$ is the set of sentences of $\mathcal{L}^2$ such that, for any ϕ in $\mathcal{L}^2$,

$$\phi\ \varepsilon\ s^{-1}(\Theta) \text{ if and only if } (\phi)^s\ \varepsilon\ \Theta.$$

Theorem: If a set T of sentences of $\mathcal{L}^1$ is a theory, so is $s^{-1}(T)$ in $\mathcal{L}^2$.

Proof: Let T be a theory in $\mathcal{L}^1$, and let Γ be a set of sentences from $\mathcal{L}^2$ and ϕ a sentence of $\mathcal{L}^2$ as well. Assume that $\Gamma \subseteq s^{-1}(T)$ and that $\Gamma \vdash \phi$ in $\mathcal{L}^2$. By definition, it follows that $\Gamma^s \subseteq T$ in $\mathcal{L}^1$. We have already demonstrated, in the preceding section, that $\Gamma^s \vdash (\phi)^s$ in $\mathcal{L}^1$. Since T is a theory, $\phi \, \varepsilon \, s^{-1}(T)$.

Definition: A theory T (in either $\mathcal{L}^1$ or $\mathcal{L}^2$) is *inconsistent* whenever there is a sentence ϕ of the relevant language such that

$$T \vdash (\phi \wedge \neg\phi).$$

A theory is *consistent* when it is not inconsistent.

Theorem: If T is a consistent theory in $\mathcal{L}^1$, $s^{-1}(T)$ is a consistent theory in $\mathcal{L}^2$.

Proof: Assume that $s^{-1}(T)$ is inconsistent. Then, by definition, for some sentence ϕ of $\mathcal{L}^2$,

$$s^{-1}(T) \vdash (\phi \wedge \neg\phi).$$

It follows from previous results and from the definition of $\phi \mapsto (\phi)^s$ that

$$T \vdash ((\phi)^s \wedge \neg(\phi)^s).$$

Therefore, T is inconsistent.

The theories of the solipsist are logically consistent if the corresponding theories in her idiolect are. Moreover, any deductive inconsistency in a solipsistic theory can be reduced, by a translation that is primitive recursive on input formulae, to one in the corresponding theory in $\mathcal{L}^1$. If the solipsist is inconsistent in her solipsism, she must already be inconsistent in her everyday views about objects and appearances, and that is not computably difficult to see. Readers who have undergone proper initiation should compare this result with that of Gödel's negative translation (Gödel 1933).

Definition: A set Γ of sentences in $\mathcal{L}^2$ is *strict* whenever $\Gamma \vdash_1 \forall x E x$.

Theorem: (Deductive Completeness) Let Γ be a strict set of sentences in $\mathcal{L}^2$ and ϕ an individual sentence. If $\Gamma \nvdash \phi$, then $\Gamma^s \nvdash (\phi)^s$.

Proof: Assume that Γ in $\mathcal{L}^2$ is strict and that $\Gamma \nvdash \phi$. Because of strictness, $\Gamma \vdash_1 E\tau$, for every term τ of $\mathcal{L}^2$. Therefore, $\vdash$ and $\vdash_1$ agree on derivations from Γ. Consequently, $\Gamma \nvdash_1 \phi$. By Henkin's Completeness Theorem (Henkin 1950), there is a model $\mathfrak{B}$ for the language $\mathcal{L}^2$ such that

$$\mathfrak{B} \vDash \Gamma \text{ and } \mathfrak{B} \vDash \neg\phi.$$

We now construct a model $\mathcal{M}$ of $\mathcal{L}^1$ from the model $\mathfrak{B}$. Take the domain $|\mathcal{M}|$ of $\mathcal{M}$ to be the very same as the domain $|\mathfrak{B}|$ of $\mathfrak{B}$. In $\mathcal{M}$, let

$$\mathcal{M} \vDash A\, R(a, b, c) \text{ just in case } \mathfrak{B} \vDash R(a, b, c),$$

for all atomic formulae $R(x, y, z)$ of $\mathcal{L}^1$, and all a, b, c in the domain or terms of the language $\mathcal{L}^2$. Also, let

$$\mathcal{M} \vDash A\, a = a,$$

for all a in the domain. Extend $\mathcal{M}$ to a model of second-order $\mathcal{L}^1$ by adding all subsets of $|\mathcal{M}|$ to constitute a domain for second-order quantification.

Plainly,

$$\mathcal{M} \vDash a = b \text{ if and only if } \mathfrak{B} \vDash a = b.$$

Because Γ is strict and $\mathfrak{B} \vDash \Gamma$, $\mathfrak{B} \vDash Ea$ for all a in its domain. $\mathcal{M} \vDash (Ea)^s$ for all a because $\mathcal{M} \vDash A\, a = a$ and $\mathcal{M}$ is an extensional model.

Assume that $\mathfrak{B} \vDash a = b$. Then, a and b are the same elements of both $|\mathfrak{B}|$ and $|\mathcal{M}|$. Therefore, $\mathcal{M} \vDash a = b$. Since $\mathcal{M}$ is extensional, $\mathcal{M} \vDash (a = b)^s$.

Conversely, assume that $\mathcal{M} \vDash (a = b)^s$. As a result,

$$A\, a = b \leftrightarrow A\, a = a.$$

Since the latter holds in $\mathcal{M}$, $A\, a = b$ holds in $\mathcal{M}$ also. By the definition of $\mathcal{M}$, $a = b$.

It then follows readily that, for all other formulae ϕ of $\mathcal{L}^2$,

$$\mathfrak{B} \vDash \phi \text{ if and only if } \mathcal{M} \vDash (\phi)^s.$$

The consequent of the theorem, namely,

$$\Gamma^s \nvdash (\phi)^s$$

is an immediate consequence.

Definitions:

1. Any theory T in either language is *axiomatized* by one of its subsets Θ just in case Θ is a recursive set of sentences and, for any sentence $\phi\, \varepsilon\, T$, $\Theta \vdash \phi$.

2. If Θ axiomatizes theory T, then one says that Θ is a set of *axioms* for T.

3. A theory T is *axiomatizable* just in case there is a subset of it serving it as a set of axioms.

Theorem: If Θ axiomatizes theory T in $\mathcal{L}^1$, while $s^{-1}(T)$ in $\mathcal{L}^2$ is strict, then $s^{-1}(T)$ is a theory and $s^{-1}(\Theta)$ axiomatizes it.

Proof: Assume that Θ axiomatizes T in $\mathcal{L}^1$ and that $s^{-1}(\Theta)$ is strict in $\mathcal{L}^2$. Assume also that ϕ is a sentence of $s^{-1}(T)$ in $\mathcal{L}^2$. By definition, it follows that $\phi^s\, \varepsilon\, T$. Since Θ is a set of axioms for T, $\Theta \vdash \phi^s$ in $\mathcal{L}^1$. By the Completeness Theorem, $s^{-1}(\Theta) \vdash \phi$ in $\mathcal{L}^2$.

Any axiomatic theory and set of axioms for that theory in the nonsolipsistic language can be carried over into the solipsistic language as a theory with corresponding axioms, provided that the latter theory is strict. Importantly, it is easy to argue – see the reply to the third interpretation of the Private Language Argument – that basic mathematical theories are all strict. Hence, a solipsist can avail herself of, say, Peano/Dedekind Arithmetic together with its familiar set of axioms.

Objection: At this point, an objector may come forward with, "You attempt to show that solipsism, as you construe it, is both coherent and defensible by assuming ahead of time that solipsism is not only coherent, but also false. In effect, you adopt an externalist position yourself, taking up a vantage point in a metalanguage and, in that metalanguage, presuming there to be another minded person, a solipsist or prospective solipsist, and that this solipsist has thoughts that intercombine, via a translation into her nonsolipsistic language, to form reasonable theories. Therefore, you are yourself an externalist *re* solipsism and, at the very least, you are begging the question: you assume that solipsism is coherent in the metatheory – but false – to show that solipsism is coherent in the object language $\mathcal{L}^2$. Second, you have left your solipsistic theory – on the present construction – in a dialectically weak position: the consistency, even the very existence, of solipsistic theories rest upon preëxisting theories in the idiolect $\mathcal{L}^1$ of which they are translations and upon the consistency of those latter theories. Hence, the supposed 'world' that $\mathcal{L}^2$ describes and in which the truths of solipsism obtain relies upon the prior existence of another 'world,' a model $\mathfrak{A}$ that $\mathcal{L}^1$ describes and in which the truths of solipsism fail to obtain."

Reply: "Allow me to respond by drawing a further analogy from research into the foundations of mathematics. The most illuminating and productive interpretations, both mathematically and philosophically, of Brouwer's mathematical intuitionism, when formalized, are Scott's topological models (Scott 1968) (and their latter-day elaborations as elementary topoi) and Kleene's realizability interpretations (Kleene 1945). For one thing, the interpretations are guarantors of relative consistency: were formalized intuitionism logically inconsistent, formalized classical mathematics, even set theory, would be as well. Although these interpretations of intuitionism get treated in many sorts of metalogics, the idea and inspiration of them are classical *au fond*: the topological and recursion-theoretic notions on which they respectively depend were discovered and developed largely, if not entirely, by classical mathematicians – not intuitionists.

However and importantly, the character and success of these interpretations, often presented as embeddings of intuitionistic universes into classical ones, do not prove that intuitionism, as founded and pursued by du Bois-Reymond, Brouwer, Heyting and others, is conceptually dependent upon classical mathematics and relies upon the latter for its very coherence. As a viable alternative to conventional mathematics, intuitionistic mathematics can be presented and extended all on its own (v. Heyting 1956) and without any assumption of the truth or coherence of classical mathematics, as its originators demonstrated. To put it another way, a mathematician can study intuitionism from within, internally, without having to adopt an external or interpretational or metamathematical perspective – classical or other – either from the outset or later, helpful interpretations notwithstanding. Those interpretations are an aid – even a great aid – to the study of mathematical intuitionism and show classical mathematicians that formalized intuitionism is metamathematically unassailable, but do not cancel the strong claim of intuitionistic mathematics to conceptual independence."

"The metageometric investigations of Eugenio Beltrami (1835–1900), Felix Klein (1849–1925), and others justify me in constructing an analogous reply to the objection by pointing to the model-theoretic study of non-Euclidean geometries, their interpretations into Euclidean models. Despite the existence of the interpretations, mathematicians know that non-Euclidean geometries can be developed internally – they call it 'synthetically' – starting from axioms in a suitable language and proving theorems in that language (von Helmholtz 1903). They need not wait upon the verdict of any interpretation. Also, non-Euclidean geometries are provably consistent relative to their Euclidean cousin."

"Solipsism can be *completely* internalized, as the long history of the subject demonstrates (cf. Carnap 1928; Goodman 1951). The fact that my constructions start from theories in $\mathcal{L}^1$ and models of those theories is inessential, an accident of exposition. As the results on derivation and axiomatization prove, one could start with $\mathcal{L}^2$ and its reasonably familiar deductive system, together with a set of axioms, and in that conjoint system, pursue the theorems of solipsism internally. Therefore, one could view and study solipsism from within, and independently of any attachment to or translation into another, nonsolipsistic language. Finally, the metamathematics reveals that any logical flaws the solipsist might uncover in the pursuit of her theories internally or synthetically must be mirrored in nonsolipsistic or everyday theorizing."

8. Testing the strong connection

It may be a drawback to the present approach that what seemed in the idea of solipsism an unbroken and general tie between existence, truth, and appearing is here broken or at least restricted. Were that strong connection undamaged, one would expect to demonstrate that

$$\phi \, \varepsilon \, S^{\mathfrak{A}} \text{ if and only if } \mathfrak{A} \vDash A\,\phi$$

(where ϕ is a formula for which the combination $A\,\phi$ makes sense) that is, what is deemed true solipsistically is precisely what, modulo the translation, appears true to the solipsist under the ordinary idiolectic circumstances captured in the standard model $\mathfrak{A}$. Unfortunately, this can be proven not to occur in various cases natural, expected, and important.

Theorem: There are models $\mathcal{M}$ of $\mathcal{L}^1$ such that $\mathcal{M} \vDash A\, \exists xRx$ but $\exists xRx \notin S$.

Proof: Let $\mathcal{M}$ be a standard frame model with underlying frame $\{\alpha, \beta\}$ such that $\leq$ is reflexive and $\alpha \leq \beta$; 0 is in the domain of the structure at α but not at β. 1 is in the domain of $\mathcal{M}$ at β but not at α. Let R be an atomic unary predicate such that $\alpha \Vdash R(0)$ only, while $\beta \Vdash R(1)$ only. The definition of forcing $\Vdash$ is the familiar one, e.g., with ϕ a formula of $\mathcal{L}^1$, w and v worlds that are either α or β,

$$w \Vdash A\, \phi$$

if and only if

$$\forall v \text{ such that } w \leq v,\ v \Vdash \phi.$$

For ϕ from $\mathcal{L}^1$, I shall say that $\mathcal{M} \vDash \phi$ whenever $\alpha \Vdash \phi$.

Consider the formula

$$\exists xRx.$$

First, note that $\mathcal{M} \vDash A\, \exists xRx$, since both α and β force $\exists xRx$.

Second, note that $\exists xRx \notin S$. For, assume to the contrary that $\exists xRx \notin S$. Then, by definition of the translation $\phi \mapsto \phi^s$, it follows that

$$\mathcal{M} \vDash \exists x((Ex)^s \wedge A\, Rx).$$

A fortiori,

$$\mathcal{M} \vDash \exists x\, A\, Rx.$$

The last, in turn, entails that

$$\alpha \Vdash A\, R0,$$

which is false.

The model $\mathcal{M}$ just constructed captures a reasonable epistemic situation. The solipsist knows that there are two red-colored light bulbs 0 and 1, located in close adjacency. The bulbs flash red very rapidly and their flashes alternate over time. 0 flashes red for brief instant, then goes dark, while 1 flashes red. When 1 is red, 0 is dark. Never is it true that both 0 and 1 are red simultaneously. World α in $\mathcal{M}$ represents specifically a situation in which 0 is red while 1 is dark, while β presents 1 red with 0 dark. The alternation is so rapid and the bulbs so near one another that normal vision cannot discern the flashes of 0 and tell them apart from those of 1. To the unaided eye, the flashes meld into the uninterrupted glow of what seems a single red light. Given such a viewing arrangement, the solipsist may well be the position to assert that $A\ \exists xRx$ – it appears to her that something is red – but be unable, given the availability of the epistemic alternative that 1 be flashing red, to claim that the bulb that shines red at any given moment is 0 rather than 1.

Nor is it the case that a strong connection between membership in $\mathcal{S}$ and appearing to the solipsist can be reinstated by adjusting for the translation $(\phi)^s$ and showing that, in general,

$$\phi\ \varepsilon\ \mathcal{S} \text{ just in case } \mathcal{M} \vDash A\ (\phi)^s.$$

Theorem: There is a model $\mathcal{M}$ of $\mathcal{L}^1$ and a sentence $(\phi)^s$ such that

$$\mathcal{M} \vDash (\neg\phi)^s$$

but

$$\mathcal{M} \nvDash A(\neg\phi)^s.$$

Proof: Consider the three-world frame $\{\alpha, \beta, \gamma\}$ with $\leq$ reflexive, $\alpha \leq \beta$, $\alpha \leq \gamma$, but no other nontrivial relational links obtaining. 0 exists at α and at γ, but not at β, where 1 exists uniquely. This time, assume that both $\alpha \Vdash R0$ and $\gamma \Vdash R0$, but that $\beta \Vdash R1$. These are the only atomic forcing conditions decorating the frame. Forcing is defined as in the preceding theorem.

Let ϕ be $R0$. On the one hand,

$$\mathcal{M} \vDash (\neg\phi)^{s} \text{ just in case } \alpha \Vdash \neg A\, R0.$$

The last is true, so $\mathcal{M} \vDash (\neg\phi)^{s}$. On the other hand,

$$\mathcal{M} \vDash A(\neg\phi)^{s}$$

fails because

$$\gamma \vDash A\, R0,$$

so that

$$\alpha \nVdash A\neg A\, R0.$$

The last model $\mathcal{M}$ represents a situation in which it does not appear to the solipsist that $A\, R0$ fails, because there is an epistemic alternative, namely γ, in which it does appear to her that 0 is red. Perhaps γ captures the situation in which the 0 bulb is lit constantly, so there is, in γ, no epistemically available alternation between bulb 0 and bulb 1 of flashing red.

Now, the connection

$$\phi\, \varepsilon\, \mathcal{S} \text{ holds if and only if } \mathcal{M} \vDash A\,(\phi)^{s}$$

obtains, according to the definition of the $\mathcal{S}$ translation, for all atomic ϕ but not in general for combinations of formula constructed using logical connectives and quantifiers, unless special conditions obtain in $\mathcal{M}$. It is important that an obvious effort to reëstablish the strong connection between truth and appearing faces a number of serious obstacles having to do with the concept of appearance. The obvious effort I have in mind consists in reformulating the original definition of $\phi \mapsto \phi^s$ to yield a new translation $\phi \mapsto \phi^A$ in which the A operator features in every clause governing logically complex formulae.

Definition: The translation $\phi \mapsto \phi^A$ is defined so that

1. $(E\tau)^A = \exists PA\ P\tau \wedge \forall P[A\ P\tau \leftrightarrow \exists x(x = \tau \wedge A\ Px)]$.
2. $(\tau = \sigma)^A = \forall P[A\ P\tau \leftrightarrow A\ P\sigma]$.
3. For any atomic formula Rxy other than assertions of existence or identity,

$$(R\tau\sigma)^A = A\ R\tau\sigma.$$

4. $(\phi \wedge \psi)^A = A((\phi)^A \wedge (\psi)^A)$.
5. $(\neg\phi)^A = A\neg(\phi)^A$.
6. $(\forall x\phi(x))^A = A\forall x((Ex)^A \rightarrow [\phi(x)]^A)$.

The first three clauses of the translation remain unchanged to respect to $(\phi)^s$; an occurrence of the A operator is intercalated into the others. This move too fails to underwrite the intended strong connection.

Theorem: There is a model $\mathcal{M}$ of $\mathcal{L}^1$ and a formula ϕ such that, over $\mathcal{M}$, $\mathcal{M} \vDash A\ \phi$ but $\phi \notin S$.

Proof: We return to the first frame model $\mathcal{M}$ with two worlds, α and β, plus $\leq$ and forcing defined conditions as before. This time, consider the formula ϕ,

$$R0 \vee R1.$$

Under the new scheme of translation, $\phi\ \varepsilon\ S$ if and only if

$$\mathcal{M} \vDash A(A\,R0 \vee A\,R1).$$

But this is clearly false.

On the other hand,

$$\mathcal{M} \vDash A\,\phi$$

just in case

$$\mathcal{M} \vDash A(R0 \vee R1).$$

This is true since, in $\mathcal{M}$,

$$R0 \vee R1$$

holds at both α and β.

The other natural alternative for a strong connection, that $\phi\,\varepsilon\,\mathcal{S}$ whenever $\mathcal{M} \vDash A\,\phi^{A}$ and conversely, would obtain in all frame models on the condition that the relation of appearance compatibility $\leq$ be transitive. However, it would be, at least, counterintuitive to insist that, if it appears to me that ϕ, it must always be the case that that very fact – that it appears to me that ϕ – itself appears to me. In any event, some very special argument would be required to guarantee it.

One can however say something on this subject with a more general bearing: that, if the strong connection, under the original $(\phi)^{s}$ translation, were indeed to hold, then the A operator would have to commute with negation $\neg$ in the idiolect.

Theorem: Let $\mathcal{M}$ be any model of $\mathcal{L}^{1}$. Assume that, for all sentences ϕ from $\mathcal{L}^{2}$, $\phi\,\varepsilon\,S$ just in case $\mathcal{M} \vDash A\,(\phi)^{s}$ (i.e., the strong connection obtains generally). Then, the A operator commutes with negation over the range of the $(\phi)^{s}$ translation: for all ϕ from $\mathcal{L}^{2}$,

$$\mathcal{M} \vDash \neg A(\phi)^{s} \text{ if and only if } \mathcal{M} \vDash A(\neg\phi)^{s}.$$

Theorem: Given that $\phi \, \varepsilon \, S$ just in case $\mathcal{M} \vDash A\ (\phi)^s$, I get, by definition, for any suitable ϕ,

$$\mathcal{M} \vDash (\phi)^s \text{ if and only if } \mathcal{M} \vDash A(\phi)^s.$$

Under that condition,

$$\mathcal{M} \vDash A(\neg\phi)^s$$

just in case

$$\mathcal{M} \vDash (\neg\phi)^s,$$

which holds just in case

$$\mathcal{M} \vDash \neg(\phi)^s$$

by the translation. This last obtains if and only if

$$\mathcal{M} \vDash \neg A(\phi)^s,$$

again, by the assumed strong connection, applied to ϕ.

There is every reason to think that "it appears to me that" does not commute with negation, even over the range of the original translation. For example, let ϕ be a logical truth the least complex expression of which contains the same number of quantifiers and connectives as the physical universe has of atoms. No one, no less the solipsist, will ever have considered or entertained the circumstance

$$\langle \text{that } \phi \rangle.$$

Therefore, under these assumptions, $\neg A\ (\phi)^s$ will certainly be true. At the same time, $\neg A\ (\phi)^s$ will surely be false as well.

Finally, one could extend the existing connection a wee bit – from atomic to strictly *conjunctive* formulae – by enlarging the range of the derivation relation $\vdash$ over formulae of $\mathcal{L}^1$ and altering the translation scheme accordingly.

Definition: A formula ϕ of $\mathcal{L}^2$ is *conjunctive* just in case it is a conjunction of atomic formulae other than those of forms $E\tau$ and $\tau = \sigma$.

For the purposes of this paragraph, I extend $\vdash$ on $\mathcal{L}^1$ to include the rules, for ϕ and ψ conjunctive,

$$A(\phi \wedge \psi) \vdash\dashv A\,\phi, A\,\psi \text{ and}$$
$$AA\,\phi \vdash\dashv A\phi.$$

(Incidentally, neither of these new rules strikes the author as intrinically plausible candidates for rules governing "it appears to me that.") At the same time, I alter the definition of the translation $\phi \mapsto (\phi)^s$ so that

$$(\phi \wedge \psi)s = \begin{cases} A((\phi)^s \wedge (\psi)^s) & \text{if } \phi, \psi \text{ are conjunctive} \\ (\phi)^s \wedge (\psi)^s & \text{otherwise} \end{cases}$$

One can then prove, with altered translation and derivations in play, that for all Γ and ϕ from $\mathcal{L}^2$, if

$$\Gamma \vdash \phi \text{ then } (\Gamma)^s \vdash (\phi)^s$$

and that for ϕ conjunctive,

$$\phi\,\varepsilon\,S \text{ if and only if } \mathcal{M} \vDash A\,\phi.$$

The strong connection has thereby been extended – at some cost in plausibility – from atomic to conjunctive expressions.

The failure of the strong connection does offer at least one signal advantage to the solipsist: she is therefore under no logical compulsion, exerted from her position on the linkage between truth and appearance, to adopt universal verificationism, the idea that there are no truths that remain unknown to her. In particular, she can allow that there are compound and general truths $\phi \, \varepsilon \, \mathcal{S}$, the set of solipsistic truths, that are not apparent to her, i.e., such that $\mathfrak{A} \nvDash A \, \phi$ and $\mathfrak{A} \nvDash A \; (\phi)^{s}$. See the treatment of solipsism and mathematical intuitionism in the next section.

9. Solipsism and intuitionistic mathematics

Now therefore is the time to turn from a generally logical examination of solipsism to the treatment of a special topic, the permanent intellectual liaison, if any, between solipsism and mathematical intuitionism in the style and spirit of Dutch mathematician L. E. J. Brouwer (1881–1966). Although Brouwer seems not have committed himself permanently to solipsism (van Atten 2004: 76–81), philosophers of mathematics have sometimes supposed that solipsism is some kind of foundation or, at least, a natural and comfortable companion to intuitionism. I can here state with authority that my internalistic approach to solipsism does not in any way require intuitionism. The mathematics of the committed solipsist need not be that of Brouwer the intuitionist.

I begin with a remark about the logical signs of the solipsist's language. There seems to be nothing at all obliging her to abandon the familiar truth-tables plus the "only two truth-values" reading of those tables that a classical mathematician or logician would grant them. The solipsist may see no bar to admitting that negation exists as a truth function and is the very truth function it is normally deemed to be, for the solipsist can maintain that negation appears to her a truth function satisfying its conventional truth-table. Moreover, she may be willing to infer from the relevant appearings, using terms with wide scope, that there is a function $\neg$ such that it appears to her that $\neg$ satisfies the truth table familiar to freshman in logic. Next, the solipsist can prove that $\neg$, as a truth-function, is solipsistically individuated, its identity determined entirely by appearings to her codified and captured in that truth-table and its usual reading. *In fine*, the negation of the solipsist can be good ole classical negation. As emphasized earlier, even abstract facts of mathematics, e.g., the monotonic increase of a function, can be made to appear and hence, be allowed solipsistically.

Returning to Brouwerian intuitionism, I remind you that Brouwer deemed the statement

$$\text{Either } x = y \text{ or } x \neq y$$

false when quantified universally on x and y. Specifically, he claimed to be able to prove, from such suitable intuitionistic principles as the Continuity Axiom (Troelstra, van Dalen 1988: 206–217), that the following statement is false.

$$\text{For all real numbers } r \text{ and } s,\ r = s \text{ or } r \neq s.$$

However, if a and b are entities for my solipsist, and negation gets from her the treatment just outlined, the statement

$$\text{Either } a = b \text{ or } a \neq b$$

seems perfectly acceptable. It is either the case that a and b share every one of her appearings involving either a or b, or there is at least one appearing of a that is not an appearing to her of b, or conversely. This is reasoning that the solipsist can carry out in her idiolect. Moreover, she need be no verificationist: there is no overall requirement on the solipsist's position that, if an arbitrary sentence of $\mathcal{L}^1$ or $\mathcal{L}^2$ be true, the solipsist knows or even suspects it to be true. The solipsistic truth of ϕ does not even imply the appearing to her of $A\,\phi$ in general, as the demonstrated failure of the strong connection shows.

Brouwer and his intuitionistic disciples reject the universal validity of the law of the excluded third, viz.,

$$\nvdash (\Theta \vee \neg\Theta).$$

One proves that, if the solipsist refuses that the law of the excluded third governs her solipsistic theorizing, then it must be refused in her nonsolipsistic idiolect. As I have shown,

$$\Gamma \vdash \phi \text{ in } \mathcal{L}^2$$

if and only if

$$\Gamma^s \vdash (\phi)^s \text{ in } \mathcal{L}^1$$

on the condition that Γ represents a strict theory. Since basic theories of mathematics such as elementary arithmetic are indeed strict, I conclude that

$$\nvdash (\phi \vee \neg\phi) \text{ in } \mathcal{L}^2$$

just in case

$$\nvdash (\phi \vee \neg\phi) \text{ in } \mathcal{L}^1,$$

for relevantly mathematical ϕ. Consequently, if the solipsist rejects the universal validity of the law of the excluded third in her solipsism, she must reject it in her idiolect. Solipsism alone seems to lend no extra boost to Brouwer's renunciation of classical logic.

10. Solipsisms for everyone

Since, up to this point, my focus has been narrowly on the *formal* logic of ontological solipsism, I have placed little weight upon the circumstance that

$$A\,\phi$$

is here standardly interpreted as

It appears to me that ϕ.

I did mention *en passant* some generalities, e.g., that A be universal on self-identities, that is, that

$$A\,a = a$$

holds for all a, and that A be veridical on identities generally, that

$$A\,a = b \rightarrow a = b$$

is true. But these were not adopted as permanent assumptions. Otherwise, nothing in the results reflects a dependence on any very specific informal construal of the A operator.

I could therefore I have set things up differently from the off. For instance, I could have thought of $A\,\phi$ as standing for

It is known to me that ϕ

or

It is known to me with certainty that ϕ

or even

It is known to me with full Cartesian certainty that ϕ.

In those cases, I would – as I proceeded through the paper – have been analyzing throughout the logic of some purified form of epistemic solipsism according to which, first, only those items that the solipsist knows to exist (or knows to exist with certainty, etc.) and are relatively independent of the solipsist's ordinary, everyday knowing are objects of quantification in her dedicated solipsistic theorizing. They would then be the gizmos satisfying the relevant *Ex* predicate; they would be the stuff existing solipsistically and epistemically. And, for atomic sentences ϕ of $\mathcal{L}^2$ that are neither *Ex* nor $x = y$, ϕ would be held true solipsistically just in case it is known to the solipsist that ϕ.

Parallel remarks apply to a moral solipsism on which $A\ \phi$ gets read as

It is morally obligatory for my sake exclusively to bring it about that ϕ.

On this understanding, the sole existents for the moral solipsist are those relatively permanent entities that play roles in the solipsist's strictly moral self-obligations.

For these styles of solipsism – plus innumerable others – the general results of the present essay always obtain. For instance, all these solipsisms, each in its own peculiar way, constitutes a complete world view in the requisite language. Moreover, there will be no refuting a consistent ontological, epistemic, or moral solipsist on her own terms: if her solipsistic views are logically inconsistent, so is her take on the nonsolipsistic universe. Over the years, some philosophers have opined that forms of committed solipsism cannot be refuted logically starting from strictly solipsistic premises. Russell (1912) wrote,

> In one sense it must be admitted that we can never *prove* the existence of things other than ourselves and our experiences. No logical absurdity results from the hypothesis that the world consists of myself and my thoughts and feelings and sensations, and that everything else is mere fancy. (22)

We are now, perhaps for the first time, in a position to *demonstrate* this insight *mathematically*.

Bibliography

van Atten, M. (2004). *On Brouwer*. Belmont: Wadsworth Publishing Company.

Carnap, R. (1928). *Der logische Aufbau der Welt* [The Logical Construction of the World]. Berlin-Schlachtensee: Weltkreis-Verlag.

Chisholm, R. (1965). The Theory of Appearing. In R., Swartz (Ed.), *Perceiving, Sensing, and Knowing* (pp. 168–186). Garden City, NY: Doubleday & Company. Reprinted from M., Black (Ed.), *Philosophical Analysis: A Collection of Essays* (pp. 102–118). Ithaca, NY: Cornell University Press.

Chomsky, N. (1968). *Language and Mind*. New York, NY: Harcourt Brace Jovanovitch.

Dedekind, R. (1888). *Was Sind und Was Sollen die Zahlen?* [What are Numbers and What are They Supposed to Do?]. Braunschweig, DE: F. Vieweg.

Dennis, L. (2012). Plan Indexing for State-Based Plans. In C. Sakama, S. Sardina, W. Vasconcelos, M. Winikoff (Eds.), *Declarative Agent Languages and Technologies IX* (pp. 3–15; Lecture Notes in Computer Science, 7169). Berlin–Heidelberg: Springer.

Descartes, R. (ed. 1979). *Meditations on First Philosophy*. Cress, D. A. (transl.). Indianapolis: Hackett Publishing Company.

Eisler, R. (1910). *Wörterbuch der philosophischen Begriffe* [Dictionary of Philosophical Concepts] (3. Aufl.). Berlin: E. S. Mittler und Sohn.

Fodor, J. (1980). Methodological Solipsism Considered as a Research Strategy in Cognitive Science. *Behavioral and Brain Sciences*, *3*, 63–73.

Gödel, K. (1933). Zur intuitionistischen Arithmetik und Zahlentheorie [On Intuitionistic Arithmetic and Number Theory]. *Ergebnisse eines mathematischen Kolloquiums* [Results of a Mathematical Colloquium], *4*, 34–38.

Goodman, N. (1951). *The Structure of Appearance*. Cambridge: Harvard University Press.

Hambourger, R. (1977). A Difficulty with the Frege-Russell Definition of Number. *The Journal of Philosophy*, *74*(7), 409–414.

von Helmholtz, H. (1903). Über den Ursprung und die Bedeutung der geometrischen Axiome [On the Origin and Significance of Geometrical Axioms]. In H., von Helmholtz, *Vorträge und Reden* [Lectures and Addresses], 2 (5. Aufl., pp. 1–31). Braunschweig: Vieweg und Sohn.

Henkin, L. (1950). Completeness in the Theory of Types. *The Journal of Symbolic Logic*, *15*(2), 81–91.

Heyting, A. (1956). *Intuitionism: An Introduction. Studies in Logic and the Foundations of Mathematics*. Amsterdam: North Holland Publishing Company.

Hilbert, D. (1926). Über das Unendlich [On the Infinite]. *Mathematische Annalen* [Annals of Mathematics], *95*, 161–190.

Kenny, A. (1973). *Wittgenstein*. Cambridge: Harvard University Press.

Kleene, S. (1945). On the Interpretation of Intuitionistic Number Theory. *The Journal of Symbolic Logic*, *10*(4), 109–124.

Kripke, S. (1982). *Wittgenstein on Rules and Private Language: An Elementary Exposition*. Cambridge: Harvard University Press.

Moore, G. E. (1925). A Defense of Common Sense. In J. H., Muirhead (Ed.), *Contemporary British Philosophy* (2nd ed., pp. 193–223). London: Allen and Unwin.

Moore, G. E. (1939). Proof of an External World. *Proceedings of the British Academy*, *25*, 273–300.

Putnam, H. (1975). The meaning of 'meaning'. *Language, Mind, and Knowledge. Minnesota Studies in the Philosophy of Science*, *7*, 131–193.

Quine, W. (1956). Quantifiers and propositional attitudes. *The Journal of Philosophy*, *53*, 177–187.

Russell, B. (1912). *The Problems of Philosophy* (Home University Library of Modern Knowledge, 35). London: Williams and Norgate.

Russell, B. (1914a). On the Nature of Acquaintance: I. Preliminary Description of Experience. *The Monist*, *24*, 1–16.

Russell, B. (1914b). The Relation of Sense-Data to Physics. *Scientia*, (4), 1–27.

Russell, B. (1923). *Human Knowledge: Its Scope and Limits*. London: George Allen and Unwin.

Russell, B. (ed. 1992). *Theory of Knowledge. The 1913 Manuscript*. Eames, E., Blackwell, K. (Eds.). London: Routledge.

von Schubert-Soldern, R. (1906). Über die Bedeutung des erkenntnistheoretischen Solipsismus und über den Begriff der Induktion, I [On the Significance of Epistemological Solipsism and on the Concept of Induction, I]. *Vierteljahrsschrift für wissenschaftliche Philosophie und Soziologie* [Quarterly for Scientific Philosophy and Sociology], *30*, 49–71.

Scott, D. (1968). Extending the Topological Interpretation to Intuitionistic Analysis. *Compositio Mathematica*, *20*, 194–210.

Scott, D. (1979). Identity and Existence in Intuitionistic Logic. In M. P. Fourman, C. J. Mulvey, D. S. Scott (Eds.), *Applications of Sheaves: Proceedings, Durham 1977* (pp. 660–696; Springer Lecture Notes in Mathematics, 753). Berlin–Heidelberg–New York: Springer-Verlag.

Todd, W. (1968). *Analytical Solipsism*. The Hague: Martinus Nijhoff.

Troelstra, A., van Dalen, D. (1988). *Constructivism in Mathematics: An Introduction*, 1 (Studies in Logic and the Foundations of Mathematics, 121). Amsterdam: North Holland Publishing Company.

Trybus, A. (2020). Two of a Kind: Setting the Record Straight on Russell's Exchange with Ladd-Franklin on Solipsism. *Russell: The Journal of Bertrand Russell Studies. New Series*, *39*(2), 101–120.

Wittgenstein, L. (1953). *Philosophical Investigations*. Anscombe, G. (transl.). New York: Macmillan.